AEROVISCOELASTIK VIBRATIONS AND FLUTTER OF SHELLS

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Abstract. In the paper, the parametric sensitivity analysis in the problem of flutter of viscoelastic cylindrical shells, with an arbitrary difference function of relaxation, is examined by the Laplace integral transform method. The critical value of free stream velocities and vibrations frequencies are determined from the condition that the real parts of the poles in Bromwich integral must be zero, which corresponds to the harmonic motion. Exact value of critical speed and corresponding frequency for a general isotropic viscoelastic constitutive relation with constant Poisson ratio are obtained. The solutions are analyzed for critical, subcritical and supercritical cases. The limit cases for short and long time are analyzed. Influence of aerodynamical damper is studied assuming the parameter of viscous property of material is smaller enough in comparison with the parameter of aerodynamical damper and vice versa.

Keywords: aero-viscoelasticity, arbitrary function of relaxation, exact solutions, damping, stability, critical speed, Laplace transformation.

AMS Subject Classification: 74H10, 74H55, 74K20

1. Introduction

There are a lot of solution methods of flutter problem in literature. The first investigation in this field is carried out in [1], which shows that the viscoelastic flutter speed may be larger or smaller than the corresponding elastic one. The similar result is also obtained in [2-6]. In [7] it is shown that the consideration of piezoelectric and viscoelastic effects does not only decrease the probabilities of failure, but also leads their occurrence in time and decreases the influence of aerodynamic noise on structures. In [8], the flutter analysis is carried out in the complex plane and calculatory computerized iterative method for the determination of flutter speeds and frequencies. The influence of viscoelastic material properties (storage and loss module) is evaluated. A new approach for an elastic flutter problems is discussed in [9]. Vibrations and asymptotical stability of viscoelastic strip for an exponential kernel of relaxation are investigated in [10]. It is shown that the critical speed is the same as an elastic strip, but viscoelastic properties of material influence the character of motion only in a subcritical domain.

A new approach for solution of flutter problems for viscoelastic materials with any relaxation functions in the stress strain constitutive relations is proposed in the current work. Using the Bubnov-Galerkin method the problem of flutter of viscoelastic shells is reduced to the solution of a certain system of integro-differential equations with appropriate initial conditions. Taking into account only first two equations of this system, the problem is solved by means of Laplace integral transform method using the method of contour integration and convolution of functions. The calculation of contour integral is usually accomplished through the use of residue theory. For this reason it is necessary to know the poles and the branch points of integrand after considering analytical continuation of the complex plane. In order to obtain the poles for any kernel of relaxation we suppose the viscous resistance of viscoelastic material is smaller than the elastic one, which is satisfied for all viscoelastic solids. The real and imaginary parts of poles, which corresponds to the damped coefficients and frequencies of vibrations respectively, are obtained.

Preliminarily, these results are obtained for the constant Poisson ratio. In [11-15] the solution technique systematizes the solution of wide class of viscoelastic problems for any functions of relaxations. Using [11-15] the obtained results in this research are generalized for all possible cases.

2. Statement of the Problem

Let us consider the problem of moving in a gas with a supersonic speed V of viscoelastic cylindrical shell which occupies the domain $0 \le \alpha \le \alpha_1, 0 \le \beta \le \beta_1$ in the $\alpha 0 \beta$ coordinate system, where α, β - dimensionless coordinates (α changes along the generator and β is the central angl) on the shell. Lateral vibrations of the shell appear during this action. After a certain value of velocity V the amplitudes of these vibrations increase without bounds for $t \to \infty$. The flutter problem consists of finding the minimum value of these velocities which is called critical.

If the Poisson ratio is constant, the problem reduces to the equation

$$c^2\Delta^4\Phi + \frac{\partial^4\Phi}{\partial\alpha^4} - \varepsilon\int_0^t \Gamma(t-\tau)\left[c^2\Delta^4\Phi + \frac{\partial^4\Phi}{\partial\alpha^4}\right]d\tau +$$
$$\frac{R}{E}\left[R\rho\frac{\partial^2}{\partial t^2} - \frac{BV}{h}\frac{\partial}{\partial\alpha} + \frac{B_1 R}{h}\frac{\partial}{\partial t}\right]\Delta^2\Phi = 0, \qquad (1)$$

where $\Delta = \frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\beta^2}$ is Laplacian, $c^2 = \frac{h^2}{12R^2\left(1-\nu^2\right)}$, $B = \frac{p_0\kappa}{v_0}$, h - thikness of shell, E - elastisity modulus, ν - Poisson coefficients, ρ - dencity, B_1 - damper coefficient, p_0 and c_0 are pressure and sound velocity in gas in infinity, κ - polytropic exponent of gas, $\varepsilon\Gamma(t)$ - kernel of relaxation, and $\Phi(t,\alpha,\beta)$ is the function in which

$$u=\frac{\partial^3\Phi}{\partial\alpha\partial\beta^2}-\nu\frac{\partial^3\Phi}{\partial\alpha^3};\quad v=-\left[\frac{\partial^3\Phi}{\partial\beta^3}+\left(2+\nu\right)\frac{\partial^3\Phi}{\partial\alpha^2\partial\beta}\right];\quad w=\Delta^2\Phi\,, \tag{2}$$

where u,v,w are the coordinates of displacement vector of the shell.

In the first part of our investigation we will consider a viscoelastic solid for which the kernel of relaxation satisfies the conditions

$$0<\varepsilon\int_0^t\Gamma(s)ds<1$$

for any t. For this reason we will assume ε to be a small positive parameter.

Equation (1) will be solved by zero initial and the following boundary conditions

$$\Phi=0,\quad \frac{\partial^2\Phi}{\partial\alpha^2}=0\quad \text{for}\quad \alpha=0\quad \text{and for}\quad \alpha=1\,,$$

$$\Phi=0,\quad \frac{\partial^2\Phi}{\partial\beta^2}=0\quad \text{for}\quad \beta=0\quad \text{and for}\quad \beta=1\,.$$

This boundary-value problem has a trivial solution $\Phi\left(t,\,\alpha,\,\beta\right)\equiv 0$, satisfying zero initial conditions. The flutter problem consists of finding the least velocity, called critical, after exceeding of which the trivial solution becomes instable.

3. Solution for small viscosity

We search for the solution of equation (1) by using Bubnov-Galyorkin method. If $\left\{\varphi_{ik}\left(\alpha,\beta\right)\right\}$ is the full set of coordinate functions, satisfying the boundary conditions, solution may be represented by the series

$$\Phi=\sum_{i,k}f_{ik}\left(t\right)\varphi_{ik}\left(\alpha,\beta\right). \tag{3}$$

Substituting this expression for (1) and requiring the result as ortogonal to all φ_{jn}, we find the system of ordinary integro-differential equations for $f_{ik}\left(t\right)$. If only two terms in (3) are taken into account for the simply supported shell as

$$\Phi\left(\alpha,\beta,t\right)=\left[\;f_1\left(t\right)\sin\frac{n\pi\alpha}{\alpha_1}+f_2\left(t\right)\sin\frac{m\pi\alpha}{\alpha_1}\right]\sin\frac{k\pi\beta}{\beta_1}\quad\left(m>n;\quad m,n,k=1,2,3,\cdots\right), \tag{4}$$

the following system of ordinary integro-differential equations is obtained

$$f_1''+a_nf_1+b\gamma f_2+2N\;f_1'=\varepsilon a_n\int_0^t\Gamma\left(t-\tau\right)f_1\left(\tau\right)d\tau\,,$$

$$f_2''+a_mf_2-\left(\frac{b}{\gamma}\right)f_1+2N\;f_2'=\varepsilon a_m\int_0^t\Gamma\left(t-\tau\right)f_2\left(\tau\right)d\tau\,, \tag{5}$$

where

$$a_i = \frac{c^2\left[\left[\left(\frac{i\pi}{\alpha_1}\right)^2 + \left(\frac{k\pi}{\beta_1}\right)^2\right]^4 - \left(\frac{i\pi}{\alpha_1}\right)^4\right]}{R^2\rho\left[\left(\frac{i\pi}{\alpha_1}\right)^2 + \left(\frac{k\pi}{\beta_1}\right)^2\right]^2}E; \quad b = \frac{4mnV}{hR\rho(m^2 - n^2)}; \qquad N = \frac{B_1}{2h\rho};$$

$$\gamma = \left[\left(\frac{m\pi}{\alpha_1}\right)^2 + \left(\frac{k\pi}{\beta_1}\right)^2\right]^2 \Big/ \left[\left(\frac{n\pi}{\alpha_1}\right)^2 + \left(\frac{k\pi}{\beta_1}\right)^2\right]^2 .$$

If we put $f_k = \exp(-N\,t)u_k(t)$ we will find following system of the two equations for the functions u_k

$$u_k'' + \left(a_k - N^2\right)u_k - (-1)^k \chi_k b u_m = \varepsilon a_k \int_0^t \Gamma(t-\tau)\exp[N(t-\tau)]u_k(\tau)d\tau , \qquad (6)$$

$$k, m = 1,2; \quad k \neq m .$$

where $\chi_1 = \gamma, \quad \gamma_2 = 1/\gamma$. Using the Laplace transformation we obtain the system of algebraic equations for the images $\bar{u}_k$ of the function u_k respectively and solve it, we will find

$$\bar{u}_1 = \frac{M_2\psi_1 - \gamma b\psi_2}{M_1M_2 + b^2}; \quad \bar{u}_2 = \frac{M_1\psi_2 + b\psi_1/\gamma}{M_1M_2 + b^2}, \qquad (7)$$

where

$$M_i = s^2 + a_i\left(1 - \varepsilon\hat{\Gamma}(s)\right) - N^2; \quad \psi_i = su_i(0) + u_i'(0),$$

$u_i(0)$ and $u_i'(0)$ are the initial values of the function $u_i(t)$ and its derivatives respectively.

In these formulas s is the complex parameter of transformation, $\hat{\Gamma}(s)$ is the Laplace transform of the multiplication $\Gamma(t)\exp(Nt)$. Assume Laplace transforms $\bar{u}_i(s)$ are analytic functions in the whole complex s-plane except for isolated singular points.

The inverse transformations of functions (7) can be found by using the contour integral method and the residue theory. For this reason we should know the poles and the branch points of integrand, having been analytically continued to the whole s-plane. To get this it is necessary to know the dependence of the function $\overline{\Gamma}(s)$ on s, which is equivalent to specify analytic expression of relaxation kernel $\Gamma(t)$. For the simple $\Gamma(t)$, which corresponds to the simple relations between strain and stress, these integrals can be calculated. The contour integral used here becomes very difficult even in case of the smallest complications of $\overline{\Gamma}(s)$ on s. Therefore the method of contour integration becomes unfit for more real relations between stresses and strains. The simulation by means of a limited set of elements unnecesserily impairs the freedom and generality of description of the behaviour of real materials and imposes restrictions that do not at all follow the fundamental laws of nature. Here we reduce a new solution method of indicated problems which completely excludes the above meantioned difficulties.

Poles of the functions (7) are the roots of the equation

$$\left(s^2 - N^2\right)^2 + \left(s^2 - N^2\right)\left(a_1 + a_2\right)\left(1 - \varepsilon\hat{\Gamma}\right) + a_1 a_2 \left(1 - \varepsilon\hat{\Gamma}\right)^2 + b^2 = 0. \tag{8}$$

Now let us define the critical value of parameter b from the equation (8). We will have to seek below at least one of the roots of the equation (8) in the form of $s = i\lambda$, where λ is a real (positive) number, i.e. the vibrations of the shell must be harmonical. For $N = 0$ we obtain two algebraic equations from (8):

$$\varepsilon\Gamma_s\left[-\lambda^2\left(a_1 + a_2\right) + 2a_1 a_2\left(1 - \varepsilon\Gamma_c\right)\right] = 0, \tag{9}$$

$$\lambda^4 - \lambda^2\left(a_1 + a_2\right)\left(1 - \varepsilon\Gamma_c\right) + a_1 a_2\left[\left(1 - \varepsilon\Gamma_c\right)^2 - \varepsilon^2\Gamma_s^2\right] + b^2 = 0, \tag{10}$$

where Γ_c and Γ_s are the Fouryer transforms of the function $\Gamma(t)$.
Equation (9) gives

$$\lambda_1^2 = \frac{2a_1 a_2}{a_1 + a_2}\left(1 - \varepsilon\Gamma_c\right). \tag{11}$$

This equation defines the root of system (9), (10) as an implicit function. Substitution of this value for equation (10) gives

$$b_{cr}^2 = \left(b^{**}\right)^2\left(1 - \varepsilon\Gamma_c\right)^2 + \varepsilon^2\Gamma_s^2 a_1 a_2 \tag{12}$$

where $b^{**} = \dfrac{a_2 - a_1}{a_2 + a_1}\sqrt{a_1 a_2}$.

These are squares of the exact critical values of frequency and parameter b. Here Γ_c and Γ_s depend on λ_1.

Substituting (12) for (10) and then deviding the result to the difference $\lambda^2 - \lambda_1^2$ it gives

$$\lambda_2^2 = \frac{a_1^2 + a_2^2}{a_1 + a_2}\left(1 - \varepsilon\Gamma_c\right).$$

Here Γ_c depends on λ_1 as well. Thus $\lambda_1^2 / \lambda_2^2 = 2a_1 a_2 / \left(a_1^2 + a_2^2\right) < 1$, we have $\lambda_1 < \lambda_2$. Since Γ_c and Γ_s are monotonically decreasing functions of λ, the value λ_1 is the smallest frequency of vibrations of the shell.

Using the expressions of λ_1^2, λ_2^2 and b_{cr}^2, equation (8) may be represented as

$$\left(s^2 + \lambda_1^2\right)\left[s^2 + \lambda_2^2 + \varepsilon\left(\Gamma_c - \overline{\Gamma}\right)\left(a_1 + a_2\right)\right] + a_1 a_2 \varepsilon\left(\Gamma_c - i\Gamma_s - \overline{\Gamma}\right)\left[\varepsilon\left(\Gamma_c - i\Gamma_s - \overline{\Gamma}\right) + \right.$$

.

$$+2\left(1-\varepsilon\Gamma_c\right)+2i\varepsilon\Gamma_s\ \Big]+b^2-b_{cr}^2=0\,. \tag{13}$$

Here for $b=b_{cr}$ we obtain the roots $s_{1,2}=\pm i\lambda_1$ since Γ_c and Γ_s also depend on λ_1. We approximately will find the other two roots of this equation assuming that

$$\left|\varepsilon\left(\Gamma_c\left(\lambda_2\right)-\Gamma_c\left(\lambda_1\right)\right)\right|<<1\,,$$

$$s_{3,4}=\pm i\tilde{\lambda}_2-\lambda_2\left(-\frac{1}{2}+\frac{1}{2}\sqrt{1+\varepsilon^2\Gamma_s^2\lambda_2^{-4}\left(a_1+a_2\right)^2}\right)^{1/2},$$

where $\tilde{\lambda}_2\approx\lambda_2\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\left(a_1+a_2\right)^2\lambda_2^{-4}\varepsilon^2\Gamma_s^2}}$. We see that $\tilde{\lambda}_2>\lambda_2$.

The terms corresponding to the smallest frequency λ_1 describe the nondamped harmonic vibrations, but the terms corresponding to the frequency $\tilde{\lambda}_2$ describe the damping vibrations. The motion of the shell is stable but not asymptotically stable.

The damping coefficient may by represented as

$$\delta=\frac{\varepsilon\Gamma_s\left(a_1+a_2\right)}{2\lambda_2\left(\frac{1}{2}+\frac{1}{2}\sqrt{1+\varepsilon^2\Gamma_s^2\lambda_2^{-4}\left(a_1+a_2\right)^2}\right)^{1/2}}\,.$$

Now let us consider the case $\left|b^2-b_{cr}^2\right|<<1$. By the method of small parameter we find the approximate value of the root of equation (27)

$$s\approx i\left[\lambda_1+\frac{b^2-b_{cr}^2}{2\lambda_1\left(\lambda_2^2-\lambda_1^2\right)}\right]+\frac{\varepsilon\Gamma_s\left(b^2-b_{cr}^2\right)}{2\lambda_1\left(\lambda_2^2-\lambda_1^2\right)^2}\left(a_1+a_2\right)\cdot$$

As we see, if $b\ =b_{cr}$ the harmonic vibrations with the frequency λ_1 take place. If $b\ >b_{cr}$ the frequency of vibrations increases proportionally to $b^2-b_{cr}^2$ and the motion of the shell becomes instable. The amplitudes of vibrations increase exponentially and proportionally to the product $\varepsilon\Gamma_s\left(b^2-b_{cr}^2\right)$. That is, for $V>V_{cr}$ the viscous resistance of material renders destabilization influence to the motion of the shell. If $b<b_{cr}$, the frequency of vibrations will decrease proportionally to $b_{cr}^2-b^2$. In this case the motion of the shell is asymptotically stable. Amplitudes of vibrations decrease exponentially and proportionally to $\varepsilon\Gamma_s\left(b_{cr}^2-b^2\right)$.

In order to obtain the simple harmonical motion for $N\neq 0$, we search for the solutions of equation (8) in form $s=N\pm i\lambda$. From equation (8) we obtain the following two equations, which correspond to real and imaginary parts of (8), respectively:

$$\begin{aligned}&\lambda^4-4N^2\lambda^2-\lambda^2(a_1+a_2)(1-\varepsilon\Gamma_c)-2N\lambda\varepsilon\Gamma_s(a_1+a_2)+\\&+a_1a_2(1-\varepsilon\Gamma_c)^2-a_1a_2\varepsilon^2\Gamma_s^2+b^2=0\end{aligned} \tag{14}$$

$$\begin{aligned}&4N\lambda\left[\lambda^2-\frac{a_1+a_2}{2}(1-\varepsilon\Gamma_c)\right]+\\&+\varepsilon\Gamma_s(a_1+a_2)\left[\lambda^2-\frac{2a_1a_2}{a_1+a_2}(1-\varepsilon\Gamma_c)\right]=0.\end{aligned} \tag{15}$$

We will solve equation (15) for λ by succesive approximation method for the cases $0\le\varepsilon\Gamma_s(a_1+a_2)<<N\lambda$ and $0\le N\lambda<<\varepsilon\Gamma_s(a_1+a_2)$, assuming Γ_c and Γ_s are constants.

a) If $0\le\varepsilon\Gamma_s(a_1+a_2)<<N\lambda$ we will find

$$\lambda_{1N}^2\approx\frac{a_1+a_2}{2}(1-\varepsilon\Gamma_c)-\frac{\varepsilon\Gamma_s(a_2-a_1)^2}{4N\sqrt{2(a_1+a_2)}}\sqrt{1-\varepsilon\Gamma_c}\,. \tag{16}$$

As we see, the critical frequency decreases while increasing ε and increases while increasing N. Substituting this expression for (14) we obtain the equation for the critical value of parameter b

$$b_{crN}^2\approx\frac{(a_2-a_1)^2}{4}(1-\varepsilon\Gamma_c)^2+4N^2\lambda^2{}_{1N}+2\varepsilon\Gamma_sN\lambda_{1N}(a_1+a_2).$$

b) If $0\le N\lambda<<\varepsilon\Gamma_s(a_1+a_2)$ then we find

$$\tilde{\lambda}_{1N}^2\approx\frac{2a_1a_2}{a_1+a_2}(1-\varepsilon\Gamma_c)+\frac{N(a_2-a_1)^2}{\varepsilon\Gamma_sa_1a_2(a_1+a_2)}\left(\frac{2a_1a_2}{a_2+a_1}(1-\varepsilon\Gamma_c)\right)^{3/2}. \tag{17}$$

Substituting this expression for equation (14) we obtain

$$\begin{aligned}&\tilde{b}_{crN}^2\approx a_1a_2\left(\frac{a_2-a_1}{a_2+a_1}\right)^2(1-\varepsilon\Gamma_c)^2+2N\varepsilon\Gamma_s\,\tilde{\lambda}_{1N}(a_2+a_1)+a_1a_2\varepsilon^2\Gamma_s^2+\\&+\frac{N\tilde{\lambda}_{1N}^3}{\varepsilon\Gamma_sa_1a_2}\frac{(a_2-a_1)^4}{(a_2+a_1)^2}\,(1-\varepsilon\Gamma_c).\end{aligned}$$

Since $4a_1a_2/(a_1+a_2)^2<1$ the frequency obtained in the formula (16) is greater than the frequency described by formula (17). The values $b^*=(a_1+a_2)/2$ and $(a_2-a_1)/2$ correspond to the square of frequency and parameter b for the elastic shells.

4. Generalization for non constant Poisson ratio

We use the elastic-viscoelastic correspondence principle between the Laplace-Carson transforms of elastic and viscoelastic problems [13, 15, 16]. If we apply the transformation to equation (1) for an elastic plate $(\varepsilon = 0)$, in the left hand side we will obtain $\frac{h^3}{12}\frac{E}{1-\nu^2}\frac{\partial^4 \bar{w}}{\partial x^4}$. So for viscoelastic plate we should invert the function as $\frac{\bar{E}}{1-\bar{\nu}^2}\bar{f}$, where

$$\bar{E} = \frac{9\bar{G}\bar{K}}{\bar{G}+3\bar{K}}, \quad \bar{\nu} = \frac{3\bar{K}-2\bar{G}}{2\left(\bar{G}+3\bar{K}\right)}.$$

Here $\bar{G}$ and $\bar{K}$ are the Laplace-Carson transforms of the shear and dilatation relaxation functions $G(t)$ and $K(t)$, respectively. It is easy to get

$$\frac{\bar{E}}{1-\bar{\nu}^2} = \frac{4\bar{G}\left(\bar{G}+3\bar{K}\right)}{3\bar{K}+4\bar{G}} = \bar{G} + 3\bar{G}\bar{g}_2 = \bar{N}$$

where $g_2(t)$ is the Iliushin greep function with the initial value $g_2(0) = (1+\nu)/3(1-\nu)$. Here G, ν and K denote the initial value of the functions $G(t), \nu(t)$ and $K(t)$, respectively. The function $g_2(t)$ can be obtained both theoretically and directly by the experiment. Let $\bar{\omega} = 2\bar{G}/3\bar{K} = (1-2\bar{\nu})/(1+\bar{\nu})$, then we have $\omega(t) = \frac{2}{3K}G(t) + \frac{2}{3}\int_0^t G(t-\tau)\,dJ_1(\tau)$ and if $\bar{K} \equiv K$ is constant, $\omega(t) = \frac{2}{3K}G(t)$ is obtained. Here $J_1(t)$ $\left(J_1(0) = 1/K\right)$ is the dilatation greep function.

It is easy to verify the equalities $\omega_{\max} = \bar{\omega}_{\max} = \omega_0 = (1-2\nu)/(1+\nu) \approx 1/4$ and $\omega_{\min} = \bar{\omega}_{\min} = \omega_0 G(\infty)/G > 0$, here $|2\bar{\omega}| < 1$ is got. So we have the absolutely and uniformly convergent expansion in a series

$$\bar{g}_2 = \frac{1}{1+2\bar{\omega}} = 1 - 2\bar{\omega} + 4\bar{\omega}^2 - \cdots,$$

and the series expression of the function $g_2(t)$

$$g_2(t) = 1 - 2\omega(t) + 4\left[\omega_0\omega(t) + \int_0^t \omega(t-\tau)\,d\omega(\tau)\right] - \cdots.$$

The inversion

$$N(t)=E(t)/\left(1-\nu^2\right)+3\int_0^t G(t-\tau)\,dg_2(\tau)$$

and

$$\frac{\bar{E}}{1-\bar{\nu}^2}\bar{f}=\overline{Nf}\doteq N(0)\left[f(t)+\frac{1}{N(0)}\int_0^t N'(t-\tau)f(\tau)\,d\tau\right]$$

are easily obtained. It is obvious that $N(0)=E/\left(1-\nu^2\right)$. If we denote $\varepsilon\Gamma_1(t)=-N'(t)/N(0)$, then we write

$$\frac{\bar{E}}{1-\bar{\nu}^2}\bar{f}\doteq\frac{E}{1-\nu^2}\left[f(t)-\varepsilon\int_0^t\Gamma_1(t-\tau)f(\tau)\,d\tau\right].$$

Thus to write equation (1) for the time dependent Poisson ratio, the kernel $\Gamma(t)$ must be replaced by the function $\Gamma_1(t)$. If we use $\Gamma_1(t)$ instead of $\Gamma(t)$ in all above obtained results, we will have the results for non constant Poisson ratio.

If Poisson coefficient is constant, then $N(t)=2G(t)/(1-\nu)$. The relation between the kernel $\Gamma(t)$, used in equation (1), and the shear relaxation function $G(t)$ is obtained as $\Gamma(t)=-2G'(t)(1+\nu)/E$.

5. Conclusion

In this research the parametric sensitivity analysis of viscoelastic shell flutter, with an arbitrary difference function of relaxation, is examined.

Viscous resistance in a material of shell leads to decreasing the critical speed in comparison with elastic shell. If a shell moves with the speed less than critical speed V_*, viscoelastic property of material shows itself as damper, i.e. reinforces the stability of the motion of the shell. If $V=V_*$ then the shell moves with harmonic vibrations with the smallest frequency. The vibrations with greater frequency take place with the exponential decay. But if $V>V_*$, viscoelastic property renders inverse influence to the motion of the shell. The frequency of vibrations becomes greater than that of harmonic ones and the amplitudes of vibrations increase exponentially. In this meaning the speed V_* may be called critical, in spite of the case of presence of aerodynamical damper the instability of the shell in this speed is not yet begining. But viscous property of material reduces the aerodynamical damper effect. The critical value of speed V_*, which is obtained for $N=0$, is a minimum of all other values of critical speed for nonzero aerodynamical damper and when the viscous property of material is absent.

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Örtüklərin aeroözlüelastik rəqsləri və flatteri

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XÜLASƏ

İşdə inteqral Laplas çevirməsi metodu ilə ixtiyari rellaksasiya fərqi funksiyasına malik özlü silindrik lövhələrin flatterinin həssaslığı məsələsinin parametrik analizi aparılmışdır. Sərbəst axının sürətinin kritik qiyməti və tezliyin rəqsi Bromviç inteqralının polyuslarının həqiqi hissələrinin sıfıra bərabərliyi şərtindən tapılır (bu harmonik hərəkət halına uyğundur). Kritik sürətin və sabit Puasson əmsallı ümumi izotrop viskoelastik münasibət üçün uyğun tezliklərin dəqiq qiymətləri tapılmışdır. Qısa və uzun zaman üçün limit halları analiz edilmişdir. Materialların özlülük xassələrinin aerodinamik tənzimləyicilərin göstəricilərinə nəzərən kiçik olduğunu nəzərdə tutaraq aerodinamik tənzimləyicinin prosesə təsiri öyrənilir.

Açar sözlər: aeroözlüelastiklik, ixtiyari relaksasiya funksiyası, dəqiq həllər, sönmə, stabillik, kritik sürət, Laplas çevirməsi.

Аэровискоеластические вибрации и пульсации оболочек

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РЕЗЮМЕ

В этой работе методом интегрального преобразования Лапласа проведен параметрический анализ чувствительности проблемы пульсации вязкоупругих цилиндрических оболочек с произвольной функцией разницы релаксации. Критическое значение скорости свободного потока и вибрации частоты определяются из условия, что действительные части полюсов интеграла Бромвича должны равняться нулю, что соответствует гармоническому движению. Получены точное значение критической скорости и соответствующие частоты для общего изотропного вязкоупругого соотношение с постоянным коэффициентом Пуассона. Решения анализируются в критическом, докритическом и сверхкритическом случаях. Проанализированы предельные случаи на короткий и длительный временные интервалы. Изучается влияние аэродинамической заслонки предполагая, что параметр вязких свойств материала является достаточно малым по сравнению с показателем аэродинамической заслонки, и наоборот.

Ключевые слова: аэро-вязкоупругость, произвольная функция релаксации, точные решения, затухание, стабильность, критическая скорость, преобразование Лапласа.